ELEMENTARY EXERCISE

1. Let $\int_0^1 \frac{dx}{\sqrt{16+9x^2}} + \int_0^2 \frac{dx}{\sqrt{9+4x^2}} = \ln a$. Find a.

2. $\int_0^{\ell n 2} x e^{-x}\, dx$

3. $\int_1^e \left(\frac{1}{\sqrt{x \ln x}} + \sqrt{\frac{\ln x}{x}} \right) dx$

4. Given $f'(x) = \frac{\cos x}{x}$, $f\left(\frac{\pi}{2}\right) = a$, $f\left(\frac{3\pi}{2}\right) = b$. Find the value of the definite integral $\int_{\pi/2}^{3\pi/2} f(x)\, dx$.

5. $\int_{-1}^{1} \frac{x\, dx}{\sqrt{5-4x}}$

6. $\int_2^e \left(\frac{1}{\ell n\, x} - \frac{1}{\ell n^2 x} \right) dx$

7. $\int_0^{\pi/4} \frac{\sin 2x}{\sin^4 x + \cos^4 x}\, dx$

8. $\int_0^{\pi/2} \frac{\cos x\, dx}{(1+\sin x)(2+\sin x)}$

9. $\int_0^{\pi/4} \frac{\sin^2 x . \cos^2 x}{(\sin^3 x + \cos^3 x)^2}\, dx$

10. $\int_{1/3}^{3} \frac{\sin^{-1} \frac{x}{\sqrt{1+x^2}}}{x}\, dx$

11. $\int_2^3 \frac{dx}{\sqrt{(x-1)(5-x)}}$

12. $\int_{3/2}^{2} \left(\frac{x-1}{3-x} \right)^{1/2} dx$

13. $\int_0^{\pi/4} x \cos x \cos 3x\, dx$

14. $\int_0^{\pi/2} \frac{dx}{5+4\sin x}$

15. $\int_2^3 \frac{dx}{(x-1)\sqrt{x^2-2x}}$

16. $\int_0^{\pi/2} \frac{dx}{1+\cos\theta \cos x}\ \theta \in (0, \pi)$

17. $\int_0^{\frac{\ln 3}{2}} \frac{e^x + 1}{e^{2x} + 1} dx$

18. $\int_0^{\pi/4} \cos 2x \sqrt{1 - \sin 2x}\, dx$

19. $\int_0^3 \sqrt{\frac{x}{3-x}}\, dx$

20. $\int_0^{1/2} \frac{dx}{(1-2x^2)\sqrt{1-x^2}}$

21. $\int_1^2 \frac{dx}{x\left(x^4+1\right)}$

22. $\int_0^{\pi/2} \sin\phi \cos\phi \sqrt{\left(a^2 \sin^2\phi + b^2 \cos^2\phi\right)}\, d\phi$ $a \neq b$ $(a>0, b>0)$

23. (a) $\int_0^{3\pi/4} \left((1+x)\sin x + (1-x)\cos x \right) dx$

(b) $\int_{\pi/2}^{\pi} x^{\sin x} \left(1 + x \cos x . \ln x + \sin x\right) dx$

24. $\int_0^1 x\left(\tan^{-1}x\right)^2 dx$

25. Suppose that f, f′ and f′′ are continuous on [0, ln 2] and that f(0)=0, f′(0)=3, f(*l*n 2)=6, f′(*l*n 2)=4 and $\int_0^{ln2} e^{-2x}.f(x)dx = 3.$ Find the value of $\int_0^{ln2} e^{-2x}.f''(x)dx$.

26. $\int_0^1 \frac{dx}{x^2+2x\cos\alpha+1}$ where $-\pi < \alpha < \pi$

27. $\int_a^b \frac{dx}{\sqrt{1+x^2}}$ where $a = \frac{e-e^{-1}}{2}$ & b = $\frac{e^2-e^{-2}}{2}$

28. $\int_{0^+}^1 \frac{x^x(x^{2x}+1)(lnx+1)}{x^{4x}+1}dx$

29. $\int_0^1 x^5\sqrt{\frac{1+x^2}{1-x^2}}\,dx$

30. Suppose that the function f, g, f′ and g′ are continuous over [0, 1], $g(x)\neq 0$ for $x\in[0,1], f(0)=0$, $g(0)=\pi$, $f(1)=\frac{2009}{2}$ and g(1) = 1. Find the value of the definite integral

$$\int_0^1 \frac{f(x).g'(x)\left\{g^2(x)-1\right\}+f'(x).g(x)\left\{g^2(x)+1\right\}}{g^2(x)}dx$$

31. $\int_0^{\pi/4} \frac{\sin\theta+\cos\theta}{9+16\sin2\theta}d\theta$

32. $\int_0^{\pi} \theta\sin^2\theta\cos\theta\,d\theta$

33. $\int_0^{\pi/2} \frac{1+2\cos x}{(2+\cos x)^2}dx$

34. $\int_0^{\pi/2} \frac{x+\sin x}{1+\cos x}dx$

35. Let $A = \int_{3/4}^{4/3} \frac{2x^2+x+1}{x^3+x^2+x+1}dx$ then find the value of e^A.

36. $\int_0^1 \frac{2-x^2}{(x+1)\sqrt{1-x^2}}dx$

37. $\int_{-1}^1 \left(\frac{d}{dx}\left(\frac{1}{1+e^{1/x}}\right)\right)dx$

38. $\int_1^e \frac{dx}{\ln(x^xe^x)}$

39. $\int_0^{\pi}\left[\cos^2\left(\frac{3\pi}{8}-\frac{x}{4}\right)-\cos^2\left(\frac{11\pi}{8}+\frac{x}{4}\right)\right]dx$

40. If $f(\pi)=2$ & $\int_0^{\pi}(f(x)+f''(x))\sin x\,dx = 5$, then find f(0)

41. $\int_a^b \frac{|x|}{x}dx$

42. $\int_{\ell n2}^{\ell n3} f(x)dx$, where $f(x) = e^{-x}+2e^{-2x}+3e^{-3x}+....\infty$

43. $\int_0^{\pi/2} \sqrt{\frac{\sec x-\tan x}{\sec x+\tan x}}\,\frac{\text{cosec } x}{\sqrt{1+2\text{cosec } x}}dx$

44. $\int_0^1 x\,f''(x)dx$, where $f(x) = \cos(\tan^{-1}x)$

45. (a) If g(x) is the inverse of f(x) and f(x) has domain $x\in[1,5]$, where f(1) = 2 and f(5) = 10 then find the value of $\int_1^5 f(x)dx + \int_2^{10} g(y)dy$.

(b) Suppose f is continuous, f(0)=0, f(1) = 1, $f'(x) > 0$ and $\int_0^1 f(x)dx = \frac{1}{3}$. Find the value of the definite integral $\int_0^1 f^{-1}(y)dy$.